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Technical Note

A model for the thermal conductivity of unconsolidated porous media based on capillary pressure–saturation relation

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1. Introduction

Porous media are most frequently met in nature and in many engineering fields. The thermal conductivity of porous media is one of the important properties for thermal design and numerical simulations. In the past decades, many measuring results were obtained for the thermal conductivity of the saturated and dry porous media [1,2], and some models have been proposed [3–5]. Relatively, the studies on the thermal conductivity for unsaturated wet porous media are limited [6]. The usually adopted method to measure the thermal conductivity of wet unsaturated porous media is the transient heat probe method. It is confirmed that the measurement are heavily affected by the moist migration during the measurement process, especially for the lower saturation. There are different treatments to obtain the thermal conductivity from the measured data [7]. De Vries [8] and Singh et. al. [9] have proposed different models to estimate the conductivity of the unsaturated porous media. But, both of these models are not convenient to use due to so many adjustable parameters in them.

The variation of capillary pressure with saturation presents the internal structure of unconsolidated porous media and the distribution of solid, liquid and gaseous phases. Here, a new model for thermal conductivity of unconsolidated porous media has been established based on the Leverett–Lewis equation for the relationship of the capillary pressure–saturation. The predictions of the thermal conductivity for dry and saturated porous media using this model agree reasonably with the results of other models and experimental data. A relation was obtained for the thermal conductivity of the unsaturated porous media when $\lambda_l/\lambda_s \leq 0.2$.

2. Formulation

The porous media is composed of solid, liquid and gaseous phases and the solid matrix is liquid wetted. Therefore, the liquid phase always occupies the narrowest pores of the porous media and formed a concave surface between liquid and gas, as shown in Fig. 1. The capillary pressure, p_c, across the interface due to the surface tension, σ, can be expressed approximately as:

$$p_c = \frac{4\sigma \cos \theta}{D} \tag{1}$$

Here, D is the characteristic diameter of the interface between liquid and gas, θ is the contact angle between the solid and liquid surface.

The volume based density function of pore diameter distribution, $\psi(D)$, in the unit representative element

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Nomenclature

D	void channel diameter
$D_{\max}$	maximum value of D for a given porous sample
f	saturation factor of capillary pressure
k	hydraulic permeability
p_c	capillary pressure
Q	quantity of heat
s	saturation
X	effective wetting area in cavities for heat transfer
$X_{\max}$	theoretical maximum value of X
x	unified effective wetting area in cavities for heat transfer
Greek symbols	
Δp	pressure step
ΔT	temperature difference
δ	unified void channel diameter
λ	effective thermal conductivity of the unsaturated porous media
λ_{dry}	effective thermal conductivity of the dry porous media
λ_f	thermal conductivity of the fluid phase for saturated porous media
λ_g	thermal conductivity of the gas phase in unsaturated porous media
λ_l	thermal conductivity of the liquid phase in unsaturated porous media
λ_s	thermal conductivity of the solid phase for unsaturated porous media
λ_u	unified effective thermal conductivity of porous media
λ_{wet}	effective thermal conductivity of wet saturated in porous media
σ	surface tensor
ϕ	porosity

volume for the porous media can be expressed as [10]:

$$\psi(D) = \frac{p_c\phi}{D}\frac{\mathrm{d}(1-s)}{\mathrm{d}p_c} \tag{2}$$

where s is the wet saturation and ϕ is the porosity of the media. For the unconsolidated porous media, the relationship of capillary pressure–saturation can be satisfactorily described with Leverett-J function as [11].

$$p_c = \frac{\sigma\cos\theta}{\sqrt{k/\phi}}J(s) \tag{3}$$

The normalized pore size distribution of the unconsolidated porous media has been plotted in Fig. 2, with the Udell's $J(s)$ function [12]:

$$J(s) = 1.417(1-s) - 2.12(1-s)^2 + 1.263(1-s)^3 \tag{4}$$

The volume of pores having radium between D and $(D + \mathrm{d}D)$ is:

$$\mathrm{d}V = \psi(D)\,\mathrm{d}D$$

Then, the porosity of the porous media can be given as:

$$\phi = \int_0^{+\infty} \psi(D)\,\mathrm{d}D \tag{5}$$

Define a variable, X, as:

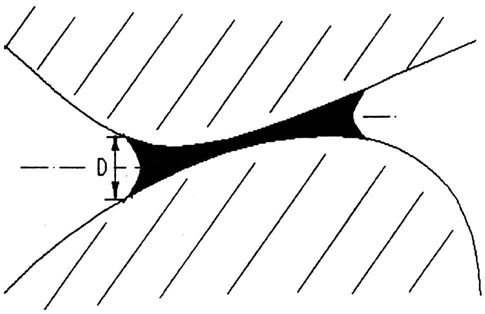

Fig. 1. Internal structure of wet porous media.

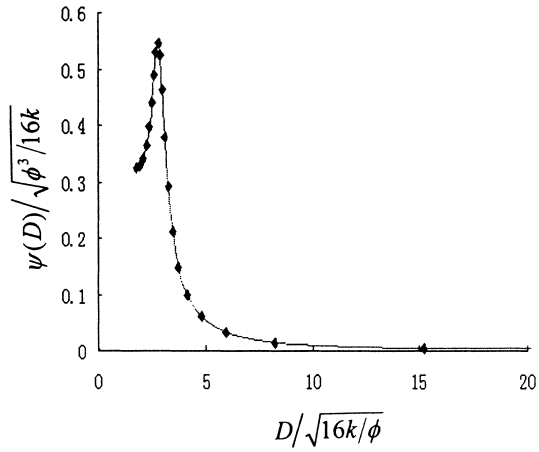

Fig. 2. Pore size distribution.

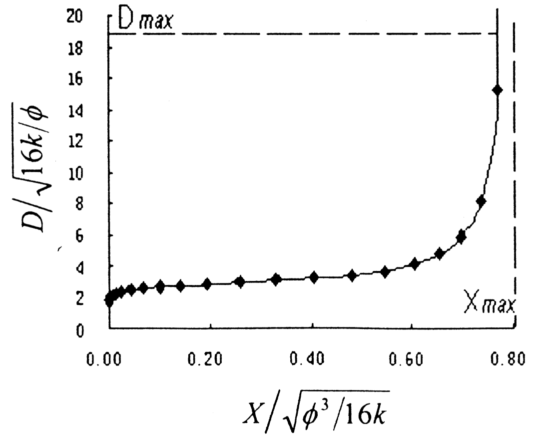

Fig. 3. Relationship between X and D.

$$X = \int_0^D \frac{\psi(D)}{D} \mathrm{d}D \tag{6}$$

The relationship between D and X is shown in Fig. 3. The upper limit of X, $X_{\max}$, can be determined with $J(s)$ function.

$$X_{\max} = 0.8024\left(\frac{\phi}{4}\sqrt{\frac{\phi}{k}}\right). \tag{7}$$

For a given porous medium, D always has a limited value. Here $D_{\max}$ may be taken as $\sqrt{20K/\phi}$ (see to Fig. 3). With normalized variables $\delta = D/D_{\max}$ and $x = X/X_{\max}$, a representative unit element was obtained in the Cartesian coordinate, as illustrated in Fig. 4. The area of zone ABCDEFHA equal to the porosity, ϕ. Since the space, whose size is less than D, is filled with water and the other space is filled with gas, X presents the effective wetting volume in pore space of the porous media. The line segment HC in Fig. 4 presents the dimensionless void size that the water can reach at certain saturation. Therefore,

$$\phi = \int_0^1 \delta(x)\,\mathrm{d}x \tag{8}$$

$$s = \phi \int_0^{x(s)} \delta(x)\,\mathrm{d}x \tag{9}$$

For a given s, δ and x can be calculated accordingly as:

$$\delta = C_1 \frac{1}{f(s)} \tag{10}$$

$$x = C_2 \int f(s)\,\mathrm{d}s \tag{11}$$

Thus, the curve shape, which presents the relationship between x and δ in Fig. 4, can be determined. The constants C_1 and C_2 are determined as follows. For porous media made up of spherical particles, the minimum porosity is 0.259 [11]. The solid particles contact closely at this situation. $\delta(0) = 0$, when $\phi = 0.259$. The integration area under the δ–x curve will be 0.259. This curve is referred as the basic one of the variation of $\delta(x)$. With the increase of ϕ, the curve $\delta(x)$ will move from right to left to ensure the sum of the area of zone II and III equals to ϕ. This means that the contact condition among solid matrix becomes weaker with the increase of ϕ. The curve $\delta(x)$ vs. ϕ is shown in Fig. 5.

For a porous medium of porosity of ϕ, the position of line segment CH varies with saturation. Thereby, the representative unit domain could be divided into three regions in Fig. 4, i.e. solid matrix volume (zone I), liquid volume (zone II) and gas volume (zone III).

In order to obtain the effective conductivity of the wet porous media, a temperature gradient was applied

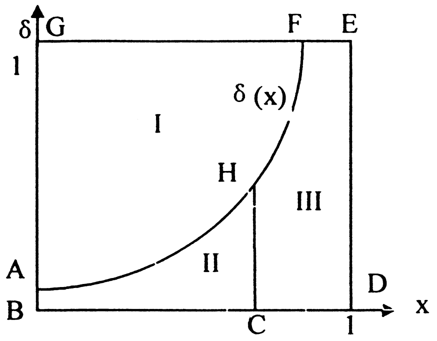

Fig. 4. Characteristic unit element for porous media with the distribution of solid, liquid and gas.

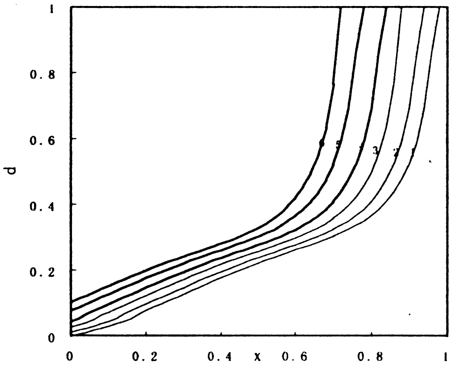

Fig. 5. Curve $\delta(x)$ and different ϕ. (1) $\phi = 0.259$; (2) $2\phi = 0.3$; (3) $\phi = 0.35$; (4) $\phi = 0.4$; (5) $\phi = 0.45$; (6) $\phi = 0.5$.

to the representative unit element. The surfaces E-F-G and B-C-D in Fig. 4 are assumed to be isothermal, while the other two boundary conditions are thermally insulated. The radiation heat transfer in the porous media can be neglected, as usual. The two-dimensional steady temperature field can be calculated with the finite element method. The quantity of heat transferred, Q, across the unit element through different phase zones, with heat conductivity λ_s, λ_l and λ_g, respectively, can be determined and the effective thermal conductivity of the wet porous media can be obtained thereby.

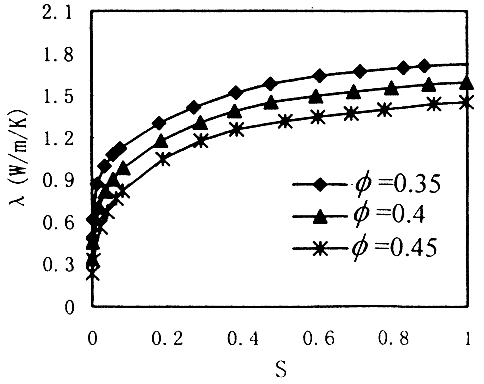

Fig. 7. The thermal conductivity at different ϕ for unsaturated porous media.

3. Results and discussions

In order to check the validity of the proposed model, the prediction for saturated or dry porous media was compared with other models, such as Kunii and Smith [3], Zehner and Schlunder [4], the Modified Z–S [5] and Nield models [13]. The results for the porous medium with $\lambda_s = 3.35$ W/m/K at saturated ($\lambda_l = 0.6$ W/m/K) and dry ($\lambda_g = 0.024$ W/m/K) conditions were shown in Fig. 6. For the dry situation, the results of the proposed model are close to that of Z–S model for larger porosity and close to that of the modified Z–S model for small porosity. At wet saturated situation, the results of present model are between the results of the Nield and Z–S models.

Table 1 showed the reasonable comparison with the experimental data given by Prasad et al. [1], in which λ_1, λ_2 and λ_3 are obtained from Zehner–Schlunder [1], Kunii and Smith [3], and Nield [13] models, respectively. λ_p is the prediction by the present model.

Noting that the effective thermal conductivity of wet unsaturated porous media is affected by porosity, wet saturation, and thermal conductivity of the solid, liquid and gaseous phases, for the porous medium of $\lambda_s = 3.35$ W/(m/K), $\lambda_l = 0.6$ W/(m/K) and $\lambda_g = 0.024$ W/(m/K), the variation of the effective thermal conductivity with saturation at different porosity is shown in Fig. 7. The effective conductivity increases very dramatically with s at low saturation. This can be explained as the result of thermal bridging of liquid phase in narrower void space at low saturation.

The effective thermal conductivity of wet unsaturated porous media of different porosity can be unified by introducing following parameter defined as

$$\lambda_u = (\lambda - \lambda_{dry})/(\lambda_{wet} - \lambda_{dry}) \tag{12}$$

Here λ_{dry} and λ_{wet} are the thermal conductivity of porous media at dry and saturated condition, which can be measured accurately. The effect of ϕ has been included in λ_{dry} and λ_{wet}. λ_u is only the function of saturation, as shown in Fig. 8. It can be fitted into following equation:

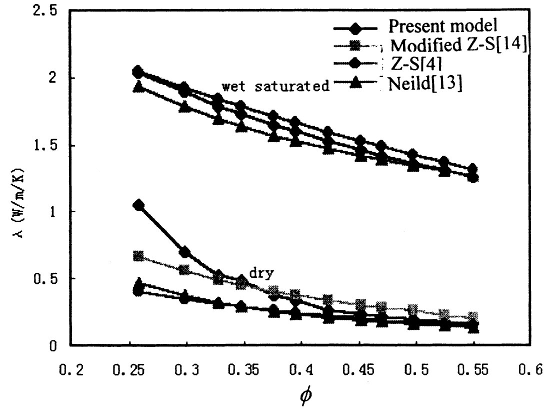

Fig. 6. The comparisons of this model with other models at dry and wet saturated condition.

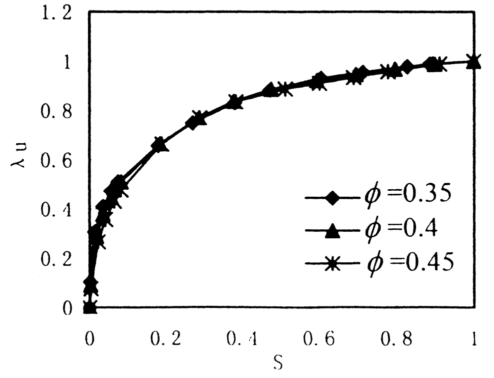

Fig. 8. The unified thermal conductivity at different ϕ for unsaturated porous media.

Table 1
The comparison with experiments

	Medium	ϕ	λ_s	λ_f	λ_1	λ_2	λ_3	Measured	λ_p
1	Water/glass	0.369	1.10	0.616	0.855	0.831	0.857	0.837	0.921
2	Water/glass	0.425	1.10	0.618	0.840	0.810	0.847	0.842	0.901
3	Glycol/glass	0.349	1.10	0.259	1.515	0.656	0.608	0.559	0.703
4	Glycol/glass	0.427	1.10	0.259	1.371	0.555	0.553	0.597	0.615
5	Glycol/steel	0.416	37.39	0.262	5.848	2.167	2.389	2.584	1.436
6	Glycol/acrylic	0.402	0.16	0.261	0.450	0.206	0.195	0.221	0.196
7	Water/acrylic	0.427	0.16	0.630	0.292	0.371	0.279	0.479	0.315

$$\lambda_u = 0.1811\ \ln(s) + 0.9878 \tag{13}$$

The effective thermal conductivity of wet unsaturated porous media can be derived as:

$$\lambda = \lambda_u(\lambda_{wet} - \lambda_{dry}) + \lambda_{dry} \tag{14}$$

4. Conclusions

In conclusion, the variation of capillary pressure with saturation reflects the internal structure of wet unsaturated porous media and the probable contribution of solid, liquid and gas phase. The proposed model may effectively predict the thermal conductivity of both saturated and wet unsaturated unconsolidated porous media. The comparisons with the experimental data for saturated porous media show that the present model is reasonable and feasible to predict the thermal conductivity of porous media. It's difficult to obtain a unified equation about the thermal conductivity of wet unsaturated porous media for all conditions. When $\lambda_l/\lambda_s < 0.2$, Eq. (14) can be used to predict the effective thermal conductivity, i.e.

$$\lambda = \lambda_u(\lambda_{wet} - \lambda_{dry}) + \lambda_{dry}$$

Acknowledgements

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